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FEBRUARY 1963 AIAA JOURNAL VOL. 1, NO. 2

Measurement of Stream Velocity in an Arc

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In electric arcs, there generally exists a considerable streaming of plasma caused by Lorentz forces generated by self-magnetic fields. In this report, the method of measurement of such streaming velocity is described. The arc used is a direct current thermal arc of 105 amp in argon at atmospheric pressure. At first the temperature of the arc was measured by a spectroscopic method and by a thermocouple, and a temperature distribution ranging from 16,000° to 1500°K was obtained. The streaming velocity was evaluated by measurement of the drag of a small plate swept across the arc, using the result of the temperature measurement, the drag coefficient of the plate, and the calculated density and viscosity coefficient of the plasma. At a section 2 mm from the cathode, the evaluated stream velocity was 135 m/sec at the center and 25 m/sec at the edge of the arc column, from cathode to anode. The error in measured temperature is estimated at less than 10%, and the error in the measured stream velocity is considered to be less than 17.5%, excluding the error due to some change of the arc discharge caused by inserting the drag measuring plate.

Introduction

INERT gas thermal arcs are used widely today as sources of high-temperature gas plasmas. However, they generally involve a considerable streaming of plasma (from cathode to anode) caused by Lorentz forces generated by the self-magnetic field. Knowledge of this streaming velocity is important to the understanding of the mechanism of arc discharge or to the use of it as a plasma source. Wienecke[1] evaluated the stream velocity in an arc from a high-speed motion picture record of the arc which was re-initiated after the current was stopped momentarily. Reed[2] obtained the stream velocity in the neighborhood of the anode of an arc by measuring total pressure through a small hole on the anode plate. In this report, a method of determination of the stream velocity in an arc by measuring the drag of a small plate, which is swept across the arc, is described. In this method it is necessary to know the temperature of the arc, and so at first a temperature measurement was made.

The Arc

The arc used is a direct current thermal arc in argon at atmospheric pressure, burning between a 3-mm-diam tungsten cathode formed into conical tip and a 20-mm-diam copper plate anode. The electrodes are arranged vertically in an arc chamber (14 cm in height and 11 cm square in cross section) and are cooled by water of fixed flow rate. In this case, the arc had axial symmetry.

In this experiment, argon of less than 0.01% impurity was bled through the chamber at a flow rate of 9.5 liter/min. The arc current was kept at 105 amp and the distance between electrodes at 5.8 mm, whereas the potential difference between them was maintained at 13.7 v. The arc discharge was stable for long-time operation, and consumption of electrode materials was unnoticed.

Temperature Measurement

The temperature in the arc column was measured spectroscopically by the method developed by Larentz[3] and Olsen,[4] and the temperature at the outside of it was measured by a thermocouple.

Measurement of Spectral Line Intensities

Spectral line intensities were measured with a spectrograph having a dispersion of 1/150 mm/Å in the infrared and a photometer consisting of a vacuum-type photoelectric cell and DuBridge circuit. The linearity of the intensity detection system was checked by changing the area of the slit of the spectrograph when lighted uniformly. The arc image was focused

Received by ARS May 1, 1962; revision received November 5, 1962.

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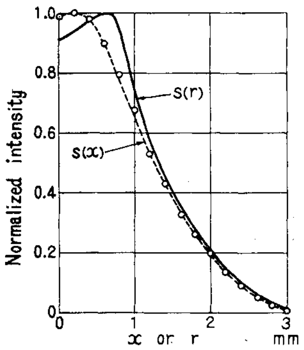

Fig. 1 **Intensity distribution of an atomic spectral line (9657.8 Å) measured at a section 2 mm from the cathode in a 105-amp, 5.8-mm atmospheric argon arc; $S(x)$ is the integrated intensity distribution, and $S(r)$ is the true radial intensity distribution**

at the slit of the spectrograph by a lens. The slit had a rectangular opening 0.2 × 0.02 mm, the longer dimension being parallel with the arc axis. In order to obtain spectral line intensity at any point in the arc column, the focusing lens was moved in its plane instead of translating the arc itself. The rotation of the system's optical axis caused by moving the lens was small (the focal length of the lens was 8 cm and the distance of its movement 1.5 mm at maximum), and with the optical arrangement of the spectrograph used here, the error caused by it was negligible. In this measurement the atomic line at 9657.8 Å, which is free from interference by neighboring lines and has intensity enough for the measurement, was selected.

The spectral line intensity distribution $S(x)$ was measured by moving the focusing lens parallel to the x axis (the origin of it is on the arc axis, and the direction of it is perpendicular to both the arc axis and optical axis of the focusing system). The dotted line in Fig. 1 shows the variation of $S(x)$ with x at the section 2 mm from the tip of the cathode (normalized to unity at maximum point). In the case of axial symmetry and negligible self-absorption from $S(x)$, which is the distribution of intensity integrated in depth, the true radial intensity distribution $S(r)$ can be obtained by a numerical method,[5] using the relation

$$S(r) = -\frac{1}{\pi}\int_r^R \frac{S'(x)\,dx}{(x^2 - r^2)^{1/2}} \tag{1}$$

where the prime denotes differentiation with respect to x, and R is radius of the arc column. The solid line in Fig. 1 shows the variation of $S(r)$ with r (normalized to unity at maximum point).

Evaluation of Temperature from Measured Spectral Line Intensities

The theoretical relation between spectral line intensity S and temperature T is given by

$$S = K \cdot [n(T)/u(T)] \cdot \exp(-E/kT) \tag{2}$$

where K is a constant for a given line, $n(T)$ the particle density of atoms in the case of an atomic line, $u(T)$ the internal partition function, k Boltzmann's constant, and E the energy of the excited state. The intensity S has a maximum value at a temperature T^*, because the value of the exponential factor increases and $n(T)$ decreases as T increases. Figure 2 shows the S-T curve for the 9657.8-Å atomic line of atmospheric argon calculated by Eq. (2), using 12.9 ev for E, and the values given in Table II of Ref. 4‡ for $n(T)$ and $u(T)$. This curve also is normalized to unity at $T = T^*$.

The temperature at the maximum point of the radial intensity distribution in Fig. 1 is T^*, and the temperature at other points can be obtained by referring to Fig. 2. The temperature distribution thus obtained is shown in Fig. 3.

‡ In Table II of Ref. 4, some correction was made on values of u_0. This caused also some variation on values of n, but S-T curve hardly was influenced by this correction.

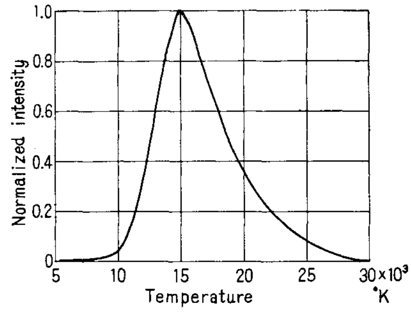

Fig. 2 **Calculated intensity-temperature relation of an atomic spectral line (9657.8 Å) of an atmospheric argon plasma**

Measurement of Temperature at Outside of Arc Column

In Fig. 3, the temperature distribution at the outside of the arc column was measured by a bare platinum/platinum-rhodium thermocouple (wire diameter is 0.5 mm) whose junction is rounded into a sphere of 1.5-mm-diam. The error due to radiation was corrected using the heat balance equation:

$$\sigma\epsilon(T_t^4 - T_0^4) = h(T_g - T_t) \tag{3}$$

where T_t is the thermocouple temperature, T_g the true gas temperature, T_0 the inside wall temperature of arc chamber, σ the Stefan-Boltzmann constant, ϵ the emissivity, and h the film heat transfer coefficient. As h is a function of the stream velocity, the evaluation of $(T_g - T_t)$ was made as follows: the stream velocity is obtained to a first approximation from the result of the drag measurement described later using T_t for the gas temperature. Using the stream velocity thus obtained, h is evaluated from the correlation of N_u and R_e for a sphere placed in an air flow,[6] and then $(T_g - T_t)$ is evaluated by Eq. (3).

Discussions of Temperature Measurement

Two assumptions are made in the spectroscopic temperature measurement. One is of local thermodynamic equilibrium, and the other is of negligible self-absorption in the arc column. The former is required for the use of Saha's equation, by which the plasma composition is calculated, shown elsewhere[4] to be allowable for atmospheric argon arcs. The latter is required for inverting the measured spectral line intensity distribution into the radial intensity distribution as previously described. The atmospheric argon arc plasma generally has been considered to be optically thin or to have negligible self-absorption, and the temperature measurement was done here using an atomic line of long wavelength (9657.8 Å). However, the absorption coefficient increases with wavelength, and in a recent report by Olsen,[7] it is shown that, when the 7635.1-Å atomic line is used for a temperature measurement of a 400-amp, 1.1-atm argon arc, a slight error (8% at maximum) is caused by the effect of self-absorption. In order to estimate the error by self-absorption in the present case, the

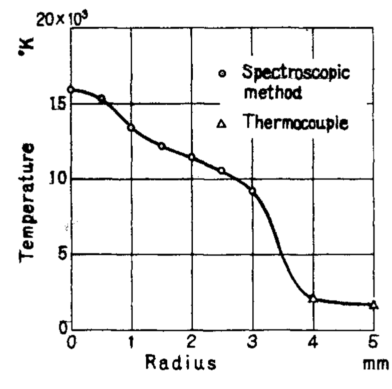

Fig. 3 **Temperature distribution at a section 2 mm from the cathode in a 105-amp, 5.8-mm atmospheric argon arc**

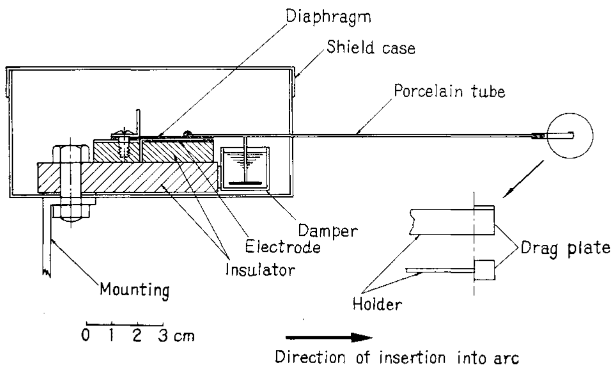

Fig. 4 Drag plate and pressure pickup of capacitance type

temperature measurement was repeated using the 7635.1-Å atomic line. In the present case, the diameter of the arc column is smaller than in the case of Olsen because of the small current, and it is expected that the error by self-absorption is reduced below 8% when the 7635.1-Å atomic line is used. The discrepancy between the temperature distribution in the case of the 7635.1-Å line and that in the case of the 9657.8-Å line was small (3.5% at maximum). The greater error in the spectroscopic temperature measurement comes from the self-absorption, and on the basis of the forementioned data it is probable that the error in the measured temperature is less than about 10%.

In the temperature measurement by the thermocouple, the correlation of $N_u - R_e$ for air is used for the evaluation of $(T_g - T_t)$. The Prandtl number of argon is somewhat smaller than that of air, and so some error will exist in the estimation of h. However, $(T_g - T_t)$ is not large compared with T_g, e.g., $(T_g - T_t) = 294°C$ and $T_g = 1724°K$ at the position $r = 5$ mm, and so the error in the measured temperature from this cause is small. Estimating the error in radiation correction at 30%, the error in measured temperature is about 6%. Since the error in the thermocouple temperature detection system is less than 4%, the total error in measured temperature will be less than 10%.

Measurement of Stream Velocity in the Arc

Measurement of Drag of Small Plate in the Arc

A square plate, 1.6 mm on a side, is inserted into the arc, keeping its surface normal to the axis of the arc. For convenience of explanation, the part of the square plate and the remainder of its support are called henceforth the drag plate and the holder, respectively. The drag plate and the holder are made of molybdenum and are attached, through an insulator, to the diaphragm of a pressure pickup of a capacitance type (Fig. 4). The sum of the drags that act on the drag plate and on some part of the holder is recorded by an oscillograph, through the pressure pickup. Since the drag plate is small and the measurement must be done quickly before the drag plate and the holder melt, the pressure pickup must meet conflicting requirements on sensitivity and response. The fundamental frequency of the diaphragm with the drag plate is about 40 cps, and the vibration of it is suppressed to some degree by an oil damper. The response of the pressure pickup is considered to be sufficient for the sweep speed used in this experiment.

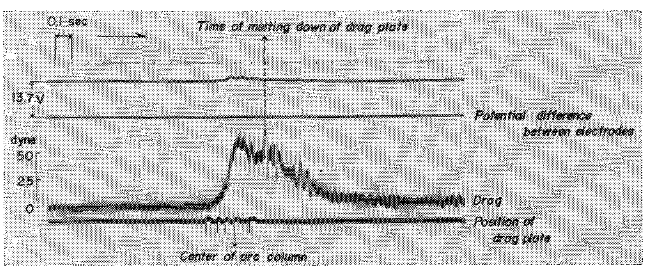

Fig. 5 Oscillogram of drag when drag plate is swept at a section 2 mm from the cathode in a 105-amp, 5.8-mm atmospheric argon arc

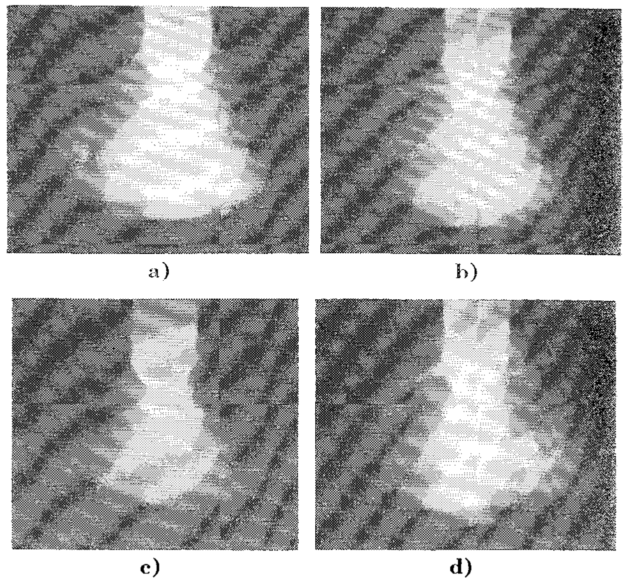

Fig. 6 Motion picture record when drag plate is swept at a section 2 mm from the cathode in a 105-amp, 5.8-mm atmospheric argon arc; time interval, 0.06 sec

Figure 5 shows the oscillogram obtained when the drag plate was swept at a section 2 mm from the tip of the cathode. The position of the drag plate in the arc is recorded, using five electrical contact points placed at intervals of 1.5 mm (the fourth point corresponds to the axis of the arc). When the drag plate is inserted, a little variation of potential difference between cathode and anode is observed in the oscillogram, although no variation was observed in the arc current. The motion picture record (Fig. 6) shows that the aspect of the arc discharge does not undergo a remarkable change by inserting the drag plate and also that the drag plate does not melt until it has passed through the center of the arc.

The net drag that acts on the drag plate at any point in the section of the arc can be obtained from the oscillogram, in comparison with a similar oscillogram obtained when the holder alone was swept at this section of the arc (Fig. 7).

Evaluation of Stream Velocity from Measured Drag

The stream velocity is calculated from the drag F by

$$F = \tfrac{1}{2}C_d\rho V^2 A \tag{4}$$

where C_d and A are the drag coefficient and the frontal area of the drag plate, respectively, and ρ is the density of the plasma.

The density of the plasma at any point can be obtained from the data of number density given in Table II of Ref. 4, since the temperature in the arc now is known.

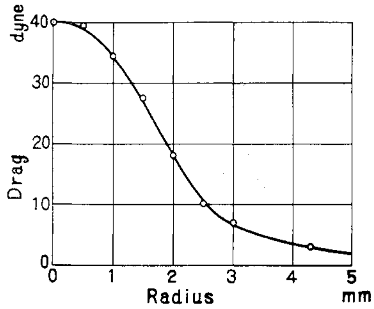

Fig. 7 Radial distribution of net drag obtained from oscillogram shown in Fig. 5

The variation of the drag coefficient of the plate with Reynolds number, related to the length of the side of the plate, is shown in Fig. 8. This result was obtained by measuring the drag of the plate in flows of oil, water, and air. In this experiment the drag was measured by a balance-type meter.

In this calculation it is necessary to know the temperature dependence of the viscosity coefficient of the plasma. The argon plasma in the arc used is ionized partially. It is known that for the viscosity coefficient of a fully ionized gas (singly charged) the role of electrons is negligible.[8] Thus, it is assumed that the viscosity coefficient of the plasma may be approximated by

$$\eta = \tfrac{1}{3}\,(n_a m_a \bar{C}_a \lambda_a + n_i m_i \bar{C}_i \lambda_i) \qquad (5)$$

In this formula, n is the number density, m the mass of a particle, $\bar{C}$ the average velocity, λ the mean free path for momentum transfer, and subscripts a and i refer, respectively, to the atom and ion.[9] It is assumed that m_a and m_i or $\bar{C}_a$ and $\bar{C}_i$ are approximately the same; λ_a and λ_i are given by

$$\lambda_a = 1/(n_a Q_{aa} + n_i Q_{ai}) \qquad (6)$$

and

$$\lambda_i = 1/(n_a Q_{ia} + n_i Q_{ii}) \qquad (7)$$

where Q is the collision cross section, and subscripts aa, ai, ia, and ii indicate the atom → atom, atom → ion, ion → atom, and ion → ion collisions, respectively. It is assumed that Q_{ai} and Q_{ia} approximately are equal to Q_{aa}. For Q_{aa}, Sutherland's formula

$$Q_{aa} = Q_{aa0}\cdot(273/T)\cdot[(T + C)/(273 + C)] \qquad (8)$$

is used, where Q_{aa0} is the cross section at 273°K, C is a constant, and T is the absolute temperature in degrees Kelvin ($Q_{aa0} = 4.3 \times 10^{-15}$ cm^2, $C = 170$ for argon). Q_{ii} is evaluated from

$$Q_{ii} = \tfrac{1}{3}\cdot m_i \bar{C}_i/\eta_i \qquad (9)$$

In this equation η_i is the theoretically obtained viscosity coefficient of a fully ionized gas, which is given by

$$\eta_i = \frac{5}{8}\cdot\frac{1}{2\{\ln(1+x^2) - [x^2/(1+x^2)]\}}\cdot\left(\frac{kT}{\pi}\right)^{1/2}\cdot\left(\frac{2kT}{e^2}\right)^2\cdot m_i^{1/2} \qquad (10)$$

where x stands for $4kT/n_i^{1/3}e^2$, and e is the charge of an electron.[8] Then, using Eq. (5) in combination with Eqs. (6–10), the temperature dependence of the viscosity coefficient of an argon plasma can be calculated (Fig. 9). In this case, for the number densities of atmospheric argon plasma (n_a and n_i), the values given in Table II of Ref. 4 were used. The decrease of the viscosity coefficient with temperature rise above 11,000°K is due to the increase of number density of ions, for which the mutual collision cross section is large.

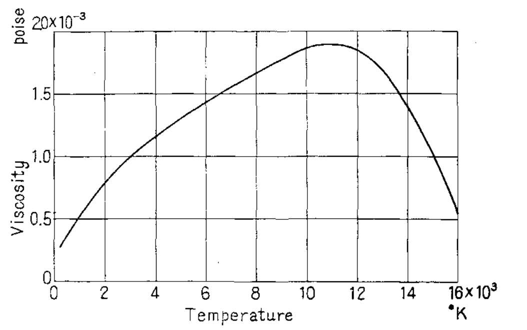

Fig. 9 Variation of viscosity coefficient of atmospheric argon with temperature

The procedure for calculation of the stream velocity at a point in the arc is as follows. From the data given in Fig. 8, the drag coefficient is expressed as a function of velocity, evaluating kinematic viscosity (contained in Reynolds number) at the arithmetic mean temperature of the plasma temperature and the estimated drag plate temperature. The drag coefficient in this form is inserted into Eq. (4), and the stream velocity then can be obtained numerically, using the value of the drag at the point given in Fig. 7. Figure 10 shows the stream velocity distributions thus obtained at the section 2 mm from the tip of the cathode. The fact that the drag plate is not sufficiently small compared to the dimension of the arc causes some error in the stream velocity distribution, especially at the central part of the arc. The corrected result, taking account of the size of the drag plate, is shown by the dotted line in Fig. 10.

Discussions of Stream Velocity Measurement

In this method, the arc discharge inevitably is influenced to some extent, electrically and aerodynamically, by the drag plate. The motion picture record (Fig. 6), however, seems to show that the variation of the arc discharge is not serious. It is assumed that the drag plate, although its electrical conductivity is larger than that of plasma, does not change substantially the electric current flow pattern in the arc, since a thermal sheath of low electrical conductivity is formed on its surface. It may be possible to decrease the influence of the drag plate on the arc discharge still more by improving the pressure pickup in sensitivity and response and by using smaller drag plates.

It is not clear which temperature is to be used for a reference temperature in the evaluation of kinematic viscosity, although in the foregoing calculation the arithmetic mean temperature is used. Kinematic viscosity varies considerably with reference temperature, but the variation of drag coefficient of the drag plate with Reynolds number is relatively small (Fig. 8),

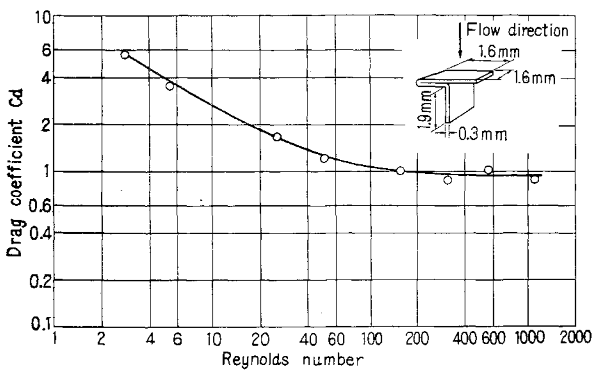

Fig. 8 Variation of drag coefficient of drag plate with Reynolds number

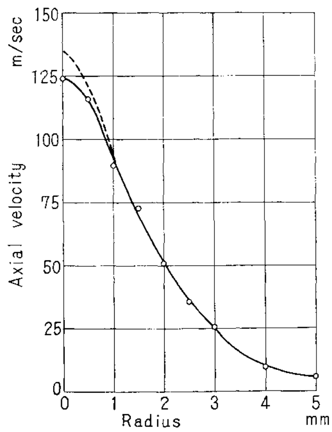

Fig. 10 Stream velocity distribution at a section 2 mm from the cathode in a 105-amp, 5.8-mm atmospheric argon arc; the dotted line shows corrected result in which size of drag plate is taken into consideration

and so the error on the stream velocity caused by the estimation of reference temperature is small. For instance, the stream velocity at the center of the arc is given as 124 m/sec when the arithmetic mean temperature 9000°K is used for a reference temperature, and the variation of this value is less than 10% when any temperature in the range from 4500° to 16,000°K is used for the reference temperature.

In the high temperature gas, the mean free path is long because of the low number density and small collision cross section. The mean free path of atmospheric argon defined by $(n_a\lambda_a + n_i\lambda_i)/(n_a + n_i)$ is 4.52×10^{-4} cm at 9000°K (reference temperature used at the center of the arc), and the corresponding Knudsen number, related to the length of side of the drag plate, is 2.83×10^{-3}. From this, it is assumed that no correction for slip flow is needed.

In the arc, besides the general plasma stream that is considered in this report, there exist the movements of electrons and ions as current carriers. It is assumed that their drag on the drag plate is negligible.

This method of stream velocity measurement is applicable only where the stream is parallel to the arc axis, because C_d shown in Fig. 8 is the value in the case where the flow is perpendicular to the drag plate. It is assumed that the stream in the arc column is nearly parallel to the arc axis, except in the neighborhood of electrodes, a result shown in Ref. 1.

In the evaluation of stream velocity by Eq. (4), the error in the density of the plasma is less than 10%, and the error in drag is less than 5%, and so the error in the stream velocity from these causes is less than 7.5%. Considering that the error in stream velocity caused by the estimation of the reference temperature will be less than 10%, the combined error in stream velocity is less than 17.5%. Not included is the error due to some change of the arc discharge caused by inserting the drag measuring plate. It is difficult to assess this error, however, and considering the motion picture record (Fig. 6), it is assumed not serious.

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FEBRUARY 1963 AIAA JOURNAL VOL. 1, NO. 2

Electrical Propulsion Capabilities for Lunar Exploration

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The logical result of a successful Apollo program can be expected to be a large scale operation involving manned lunar exploration and colonization. This paper will examine the potential role of electrical propulsion in such an operation. Primary emphasis is placed on the identification of mission capabilities for providing logistic support. Electrical engine and nuclear-mechanical power conversion system experience is projected to the 1 to 10 Mw range and used to identify expected system characteristics. Both thermal arc and ion engines are considered. A basic mission profile is defined, and typical trajectory characteristics are discussed. The low thrust characteristic velocity requirement for the basic mission is identified and used to generate parametric mission performance characteristics for one-way, round-trip, and multitrip missions. These data indicate the interrelationships between gross weight, power rating, specific impulse, and total propulsion time and their effect on payload capabilities. The resulting data then are compared with the capabilities of comparable chemical and nuclear rocket-propelled vehicles.

Nomenclature

a_3 = semimajor axis of initial lunar orbit, miles
e_1 = eccentricity of earth transfer orbit
E = eccentric anomaly of earth transfer orbit, rad
f = universal gravitational constant, $9.404(10)^{-14}$ miles^3/lb-hr^2
F = engine thrust, lb
g_0 = sea level gravitational constant, 79,019 miles/hr^2
I_{sp} = engine specific impulse, sec
K_0 = $(p/fM_e)^{1/2}$, hr/mile
M_e = weight of the earth, $1.3177(10)^{25}$ lb
M_m = weight of the moon, $1.6204(10)^{23}$ lb
p_0 = semilatus rectum of initial earth satellite orbit, miles
p = semilatus rectum of earth transfer orbit, miles
r_{em} = earth-moon distance, miles
r_{ev} = earth-vehicle distance, miles
r_{mv} = moon-vehicle distance, miles

Presented at the ARS Electric Propulsion Conference, Berkeley, Calif., March 14–16, 1962; revision received November 30, 1962.

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